Stability of doubly clamped beam geometry in fracture toughness testing of small scale systems

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• Motivation and Objective: Stability of cracking

• Introduction: Doubly Clamped Beam Bend Geometry

• Method: Determination of $K_{IC}$ using stable crack growth

• Modeling: XFEM simulations

• Results: XFEM
  
  Experiments on glass and NiAl

  $K_{IC}$ across bond coat and R-curve behavior

• Conclusions
Motivation

Small scale fracture toughness testing


Espinosa, *J Microelectromech Sys*, 2005

Motivation

Stability in Fracture Toughness Testing Geometries

Unstable fracture

$X \, d\nu = d\Lambda + R \, dA + dH$

Stable fracture

Ref. Mai, TJSAED, 1980
Motivation

Stability criterion

\[ G = \left(-\frac{\partial \Lambda}{\partial A}\right)_u = \left(\frac{\partial \Omega}{\partial A}\right)_x = G_c \]

Geometric stability factor: \( \frac{dG}{dA} \)-depends on specimen geometry and mode of loading

\[ \frac{dR}{dA} \geq \left(\frac{\partial G}{\partial A}\right)_x \]

Instability can be caused by:
- Choice of specimen geometry
- Microstructural crack resistance-falling R-curve
- Blunt notches in place of sharp cracks

Ref. Mai, TJSAED, 1980
Common testing geometries and stability factor

**SENT:** Unstable

**DCB:** Stable in disp control; unstable in load control

**SENB:** Unstable except for large $a/W$ ratios in disp control

**SENB with chevron notch:**

Ref. Mai, TJSAED, 1980
Objective

Devising a new fracture toughness testing geometry for promoting stable crack growth in inherently unstable materials at the micron length scale
Introduction: Doubly Clamped Beam Bend Geometry

- Single Edge Notch Bend (SENB) geometry commonly used to determine fracture toughness of brittle systems
- SENB: Analytical $K_I$ known, unstable in load control

- Modified SENB-elastically clamped beam geometry used
- Analytical solution for $K_I$ of SENB samples cannot be directly adopted-difference in boundary condition: free ends vs fixed ends changes the geometric factor $f(a/W)$

3-point bend $y = \frac{Pl^3}{48EI}$  
Clamped beam bend $y = \frac{Pl^3}{192EI}$
Introduction: Doubly Clamped Beam Bend Geometry

20 μm

15 μm

100 μm
**Introduction:** $K_{IC}$ from Doubly Clamped Beam Geometry

**Loading:** Wedge indenter used to bend microbeam in-situ or ex-situ

Pop-in load/change in compliance correlated with crack initiation or propagation event

Increasing load taken up for continuing crack propagation-geometric effect?
Overloading failure
Introduction: $K_{IC}$ from Doubly Clamped Beam Geometry

**Modeling:** Analytical solutions not available for determination of $K_I$ for this geometry
Stress distribution unknown
Pop-in load used to calculate $K_{IC}$ from *Extended Finite Element Analysis-XFEM* in *ABAQUS*
XFEM Modeling:

XFEM analysis in **Abaqus** environment - to extract $K_I$ as a function of load and crack length

Part Assembly

**Elements:** CPE4 (Plane Strain) in 2D; hexahedral elements in 3D
Total: ~40,000 elements

**Crack domain:** Enriched XFEM elements in the domain containing crack. Maximum principal stress for failure criterion used for crack initiation - **MAXPS** = 1 GPa

**Material properties:** $E=180$ GPa, $\nu=0.25$ – elastic, **LEFM** used
Results: XFEM Modeling-Validation on Three Point Beam

Unnotched specimen

Notched specimen

\[
K_I = \frac{3\sqrt{\frac{a}{W}} \left[ 1.99 - \left( \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right) \left( 2.15 - 3.93 \frac{a}{W} + 2.7 \left( \frac{a}{W} \right)^2 \right) \right]}{2 \left( 1 + 2 \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right)^{\frac{3}{2}}} \frac{PS}{BW^{\frac{3}{2}}}
\]
Results: XFEM Modeling - Unnotched Beam

Position of maximum principal stress

3 point bending

Clamped beam bending
Gradual increase in compliance during crack propagation
Beyond \(a/W>0.8\), compressive field of indenter interferes with the opening tensile stress, bringing a transition from pure mode I-not valid
Results: XFEM Modeling-Stability

Three point beam

Clamped beam

P = 0.03 N

P = 0.32 N

P = 0.37 N
Results: XFEM Modeling-Stability

Three point beam bend geometry:  
free ends

Clamped beam bend geometry:  
fixed ends

$K_I$ falls off with increasing crack length beyond a critical $a/W$ ratio for clamped beam geometry, even in load control.
Results: XFEM Modeling-Stability

Schematic: Beam stability under load control
Results: XFEM Modeling-Stability

Stress redistribution and edge cracking

Edge cracking ensues once central crack propagates beyond $a/W > 0.75$ - shift in position of MAXPS from bottom centre to fixed edges.
Results: XFEM Modeling-Edge cracking
Results: XFEM Modeling-Edge cracking

Constant thickness: 100*20*20 at P=0.8 N

1 µm edge crack

5 µm edge crack
1. Clamped beam loaded against the notch at the centre

2. Crack initiates and propagates from the notch tip with increasing load

3. Edge cracks appear and central crack arrests – clamped beam is now like two single cantilevers loaded at the free end
Results: Experiment-Fused Silica Glass

Before loading

$K_{IC} : 1.1 \text{MPa}\cdot\text{m}^{1/2}$

Blunt notch could have driven instability
Results: Experiment-Bulk single crystal Ni$_{50}$Al$_{50}$

Continuous drop in stiffness: controlled crack growth

$K_{ic}$: 4.3 to 5.5 MPam$^{1/2}$

No edge cracking
Results: Experiment-R-curve Effect in Bond Coat

Increasing loads required for continued crack propagation
• 5PtAl shows rising fracture toughness with increasing crack growth
• Grain boundary bridging may cause toughening
CONCLUSIONS

• Doubly clamped microbeam geometry stable under load control-can be used to determine both $K_{IC}$ and R-curve behavior

• Fracture toughness found across glass, NiAl and bond coat systems

• Rising fracture toughness seen across beam width, suggests toughening mechanisms in crack wake operating in bond coat
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THANK YOU
Doubly Clamped Beam Bend Geometry

Outline:

Show schematic of the geometry and indenter-formulas for E, fracture stress and then KIC-motivation for XFEM
Determination of P-d curve and KIC from the geometry
Observation of crack arrest
FEM simulations testing for stability, edge cracking, comparison with 3Pt bending et al
Experimental verification: Glass (Macro-scale) and Bulk NiAl (Micro-scale)
Experimental results on bond coats and possibilities for R-curve measurements
Bulk NiAl-Beam 1-28mN
2PtAl-R-curve
Introduction: XFEM Modeling

SIF calculated based on interaction integrals

Contour integral around crack tip transformed to domain form of interaction energy integral

\[ I = - \int_A (\sigma_{ik} \varepsilon_{ik}^a \delta_{1j} - \sigma_{ij} u_{i,1}^a - \sigma_{ij}^a u_{i,1}) q_j dA \]

\[ I = \frac{2}{E^* \cosh^2(\pi \epsilon)} [K_1 K_1^a + K_2 K_2^a] \]

\[ K_1 = \frac{E^* \cosh^2(\pi \epsilon)}{2} I_1, \quad K_2 = \frac{E^* \cosh^2(\pi \epsilon)}{2} I_2 \]

Abaqus computes SIFs only for stationary cracks using calculated displacement field matrix at integration points of each enriched element

Cracks always propagate towards loading line—in direction of max tangential stress
Predicted crack trajectory matches well with experimentally observed one
• Mode mixity introduced due to misalignments in loading
• Mode mixity increases steadily with increasing offset

Minor misalignments do not bring about large difference in $K_I$
Microbeam bending: XFEM results

Notch root radius

P = 0.8 N; a/W = 0.375

SIFs from FIB machined notches → sharp cracks
Standard geometries providing stable cracking with stiff machines:

- Plain and tapered DCB
- Ball indentation cone cracks
- Three point beam with long starter cracks
- Double torsion specimen
- Compact tension specimen
- Peel testpiece
Results: Experiment-(Pt,Ni)Al Bond Coat

Zone 1. 2 phase PtAl₂ ppt in β-10μm

Zone 2. β-PtNiAl 50 μm

Zone 3. Ppt free zone 15 μm

Zone 4. Inter-diffusion zone 20 μm